Numerical modeling of terahertz generation via difference-frequency mixing in two-color laser

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ABSTRACT

We present the numerical study of terahertz generation via different frequency mixing in two-wavelength vertical external cavity surface emitting laser. Nonlinear crystal is placed inside the resonator to increase terahertz radiation power. The dynamical model is based on modified Lang-Kobayashi equations. Numerical simulation through varying round trip time in the external cavity and feedback rate is presented.

Keywords: Terahertz, difference-frequency, two-color laser, Lang-Kobayashi equations

1. INTRODUCTION

Terahertz (THz) radiation, which frequency range lies in the gap between 100 GHz and 30 THz, has many attractive properties.\(^1\) THz waves have low photon energies and thus cannot lead to photoionization in biological tissues as can X-rays. As a result, they are considered safe for both the samples and the operator. At THz frequencies, many molecules exhibit strong dispersion and absorption due to dipole-allowed rotational and vibrational transitions. Most kind of dry dielectric materials, such as cloth, paper, wood, and plastic are transparent to THz waves. The wavelength of the THz waves is sufficiently short to provide submillimeter level spatial resolution. All of that provide a huge opportunities for many applications such as terahertz time-domain spectroscopy,\(^2\) rotational-vibrational spectroscopy,\(^3\) safety monitoring and quality control,\(^4\) nondestructive testing,\(^5\) astronomy and atmospheric research,\(^6\) short distance wireless communications and networking,\(^7\) etc.

There are three main approaches for developing THz sources:\(^8\) optical THz generation: THz generation via quantum-cascade lasers,\(^9,10\) where emission occurs owing to electron relaxation between subbands of quantum wells; application of electronic devices, such as free-electron lasers,\(^11\) gyrotron,\(^12\) backward wave oscillator,\(^13\) which are already well established at low frequencies. Optical methods of THz generation, which are rapidly developing in the last decade, involves generating an ultrafast photocurrent in a photoconductive switch or semiconductor\(^14\) and THz emission via nonlinear optical effects such as optical parametric oscillation,\(^15\) optical rectification\(^16\) or difference-frequency generation (DFG).\(^17\)

Despite significant efforts dedicated to the development of radiation sources for THz frequency region the problem of getting compact low-cost broadly tunable high efficient emitter working at room temperature is still remaining to be thorny. THz sources based on semiconductor lasers in the couple with nonlinear crystals, in particular those using \(\chi^{(2)}\) nonlinearity, are very promising as a result of the fact that they combine broad tunability by controlling the semiconductor alloy composition, decent output power, which shows quadratic dependence vs. pump power, capability of optical scheme variations and compactness. Dual-wavelength vertical cavity surface emitting lasers (VECSEL) can perform such kind of device. In dual-wavelength VECSEL two coaxial near-diffraction limited beams are emitted simultaneously in one monolithic structure, while sharing the same optical cavity. A theoretical and experimental model of such laser was proposed by Morozov \textit{et al.}\(^18,19\) For the first view external cavity may be seemed as sidestep to more bulk construction. But this approach makes possible to achieve single-mode high power beam and mount nonlinear crystal inside the resonator. Nonlinear frequency conversion within the laser cavity can be great opportunity for increasing the THz power. This is because the fields intensities inside the cavity exceed intensities outside of the laser.

However the dynamic complexity of such type of system grows dramatically. On the one hand conventional VECSEL without nonlinear crystal demonstrates huge variety of nonlinear dynamic behaviors, including, e.g., multistability, self-pulsations and chaotic regimes.\(^20\) The presence of optical feedback from a distant mirror causes destabilization of the stationary lasing state. In experiments, the length of the external cavity can vary
from less than 1 mm in monolithically integrated devices up to 1 m in the case of reflection from a distant mirror. With increasing delay, the dynamical complexity also increases, and finally a high-dimensional chaotic behavior can be observed. On the other hand the problem of output intensity stability subject to lasers with nonlinear frequency conversion element is also very important. It was shown theoretically and experimentally that when increasing the nonlinear coupling between the modes, the laser can exhibit large amplitude oscillations of modal intensities. So the problem of dynamic analysis of dual-wavelength VECSEL with intracavity nonlinear frequency conversion has large theoretical and applied potential.

In this paper we present the dynamic model of two-color vertical external cavity surface emitting laser with nonlinear crystal inside the cavity for THz generation. Our model is based on the Lang-Kobayashi system of delayed differential equations. We present the numerical solutions, which shows the variety of system dynamics regimes, which can be used in follows for increasing THz power. This is due to the fact that efficiency of nonlinear frequency conversion is much higher in pulse regime rather than continuous wave.

2. DYNAMIC MODEL

The model of laser under study is shown on Fig. 1. The laser resonator in Z-configuration consists of semiconductor chip, two curved mirrors and the output coupler. All mirrors have high reflectivity coefficient at the fundamental wavelengths and high transmission for the difference frequency. Curved mirrors is utilized to focus fundamental waves onto nonlinear crystal for boosting conversion efficiency. The laser chip consists of different subcavities and Bragg reflector with high reflectance in near infrared region. Sophisticated design of active region allows to emit at two wavelength with fundamental modes simultaneously. For more details see Morozov et al. Wavelengths are assumed to be at 985 nm and 1042 nm respectively. For this scheme difference frequency occurs at 30 µm (10 THz). The laser is pumped with an 808 nm fiber-coupled diode laser.

Theoretical investigations of optical feedback effects are usually based on the Lang-Kobayashi rate equations which have proven to contain all dominant effects observed experimentally. The Lang-Kobayashi model is a system of delay differential equations describing the evolution of the complex electric field and excited carriers density. The presence of so-called linewidth enhancement factor α is the main reason for dynamical instability under delayed feedback. Considering the fact that in our model we have two laser generators, coupled through nonlinear crystal, the set of equation can be rewrite as

\[
\begin{align*}
\frac{d}{dt} E_s(t) &= \frac{1}{2} (1 + i\alpha) \left[ G(E_s, N_s) - \gamma_e |E_s| - G(E_s, N_s) |E_s|^2 \right] E_s(t) - \chi_e |E_s|^2 E_l(t), \\
\frac{d}{dt} N_s(t) &= \frac{1}{2} \left(1 - i\alpha\right) G(E_s, N_s) |E_s|^2, \\
\frac{d}{dt} E_l(t) &= \frac{1}{2} (1 + i\alpha) \left[ G(E_l, N_l) - \gamma_e |E_l| - G(E_l, N_l) |E_l|^2 \right] E_l(t) - \chi_e |E_l|^2 E_s(t), \\
\frac{d}{dt} N_l(t) &= \frac{1}{2} \left(1 - i\alpha\right) G(E_l, N_l) |E_l|^2, \\
E_r(t) &= \chi_r E_s(t) E_l^*(t).
\end{align*}
\]

The indexes s, l, r are referred to shorter wavelength, longer wavelength and difference frequency respectively. \( E_{s,l,r} \) – slow varying electric field amplitudes.

The feedback phase factor \( e^{-i\Omega_{s,l} \tau_{ec}} \) depends on the delay time \( \tau_{ec} \) of the external cavity. However, since \( \Omega_{s,l} \) is very large, slight changes in the delay time change the phase drastically without changing the delay term of the slowly varying envelope \( E(t - \tau) \). The gain function with saturation in our case can be presented as

\[
G(E, N) = g \frac{N - N_T}{1 + \epsilon |E|^2}
\]

The remaining variables in the equations (2) are: \( \gamma \) - photon decay rate; \( \gamma_e \) - carrier decay rate; \( \tau_{ec} \) - round-trip time in the external cavity; \( \alpha \) - alpha-factor; \( P \) - pump power; \( k \) - feedback rate; \( g \) - differential gain; \( N_T \) - carrier density.
Table 1. Parameters of the modified LK model

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Orders of magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon decay rate</td>
<td>$\gamma$</td>
<td>$10^{11}$ s$^{-1}$</td>
</tr>
<tr>
<td>Carrier decay rate</td>
<td>$\gamma_c$</td>
<td>$10^9$ s$^{-1}$</td>
</tr>
<tr>
<td>Round-trip time in the external cavity</td>
<td>$\tau_{ec}$</td>
<td>$(0.05 - 0.22) \times 10^{-9}$ s</td>
</tr>
<tr>
<td>Alpha factor</td>
<td>$\alpha$</td>
<td>4</td>
</tr>
<tr>
<td>Pump power</td>
<td>$P$</td>
<td>1.25 W</td>
</tr>
<tr>
<td>Feedback rate</td>
<td>$k$</td>
<td>$(0.01 - 0.1) \times 10^{11}$ s$^{-1}$</td>
</tr>
<tr>
<td>Differential gain</td>
<td>$g$</td>
<td>$10^4$ s$^{-1}$</td>
</tr>
<tr>
<td>Carrier number at transparency</td>
<td>$N_T$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Gain saturation coefficient</td>
<td>$\epsilon$</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>Solitary laser frequency (shorter wavelength)</td>
<td>$\Omega_s$</td>
<td>$10^{14}$ s$^{-1}$</td>
</tr>
<tr>
<td>Solitary laser frequency (longer wavelength)</td>
<td>$\Omega_l$</td>
<td>$0.9 \times 10^{14}$ s$^{-1}$</td>
</tr>
<tr>
<td>Nonlinear susceptibility (shorter wavelength)</td>
<td>$\chi_s$</td>
<td>$9.41 \times 10^{-16}$ m/V</td>
</tr>
<tr>
<td>Nonlinear susceptibility (longer wavelength)</td>
<td>$\chi_l$</td>
<td>$9.14 \times 10^{-16}$ m/V</td>
</tr>
<tr>
<td>Nonlinear susceptibility (THz wavelength)</td>
<td>$\chi_r$</td>
<td>$8.22 \times 10^{-9}$ m/V</td>
</tr>
<tr>
<td>Pump conversion ratio</td>
<td>$\zeta$</td>
<td>$10^{17}$ (sW)$^{-1}$</td>
</tr>
</tbody>
</table>

number at transparency; $\epsilon$ - gain saturation coefficient; $\Omega$ - solitary laser frequency; $\chi$ - nonlinear susceptibility. Numerical values of the variables are given in the Tab.1.

Nonlinear crystal is presented as a last term in the equation for amplitude. In respect to shorter wavelength it is responsible for energy dissipation; for longer wavelength - for energy gaining.

3. NUMERICAL RESULTS

In order to start numerical simulation we need to bring the system of equations to dimensionless form. The most common way to do that is to introduce dimensionless time $S$ and new dynamical variables which are functions of $S$ and related to the original parameters in the form:

$$S = t / t_c, E(t) = \varepsilon_c E(t / t_c), N(t) = n_c N(t / t_c) + n_c^0.$$ 

$t_c, \varepsilon_c, n_c$ are the characteristic factors, that can be determined from following conditions:

$$1 = t_c g n_c, 1 = (n_c^0 - N_T) / n_c, 1 = t_c \gamma, 1 = \frac{g}{\gamma} \varepsilon_c^2.$$ 

Solving the system gives following values of characteristic factors:

$$t_c = \frac{1}{\gamma}, \varepsilon_c = \sqrt{\frac{\gamma g}{g}}, n_c = \frac{\gamma}{g}, n_c^0 = N_T + \frac{\gamma}{g}.$$ 

Introducing the factors into equations reveals dimensionless Lang-Kobayashi equations in the following form:

$$\frac{d}{dS} E_s(S) = \frac{1}{2} (1 + \alpha) \left[ \frac{1 + N_s}{1 + \mu |E_s|^2} - 1 \right] E_s + K e^{i \varphi_s} E_s(S - \tau) - c_s E_s |E_l|^2,$$

$$\frac{d}{dS} N_s(S) = \frac{1}{T} \left( p - N_s - \frac{1 + N_s}{1 + \mu |E_s|^2} \right).$$
\[
\frac{d}{dS} E_l(S) = \frac{1}{2} (1 + \alpha) \left[ \frac{1 + N_l}{1 + \mu |E_l|^2} - 1 \right] E_l + K e^{i\varphi_l} E_l(S - \tau) + c_l E_l |E_s|^2, \\
\frac{d}{dS} N_l(S) = \frac{1}{T} \left( \rho - N_l - \frac{1 + N_l}{1 + \mu |E_l|^2} \right), \\
E_r(S) = c_r E_s(S) E_l^*(S).
\]

(3)

Table 2. Normalized parameters of the modified LK model

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time scale parameter</td>
<td>(T)</td>
<td>(\frac{\omega_e}{\gamma})</td>
</tr>
<tr>
<td>Time delay</td>
<td>(\tau)</td>
<td>(\gamma T_{ee})</td>
</tr>
<tr>
<td>Gain saturation coefficient</td>
<td>(\mu)</td>
<td>(\frac{\gamma e}{\lambda e})</td>
</tr>
<tr>
<td>Feedback rate</td>
<td>(K)</td>
<td>(\frac{\kappa}{\gamma})</td>
</tr>
<tr>
<td>Pump rate</td>
<td>(p)</td>
<td>(\frac{\Omega}{\gamma e} - N_T) - 1</td>
</tr>
<tr>
<td>Alpha factor</td>
<td>(\alpha)</td>
<td>4</td>
</tr>
<tr>
<td>Feedback phase (shorter wavelength)</td>
<td>(\varphi_s)</td>
<td>(-\Omega_{ee})</td>
</tr>
<tr>
<td>Feedback phase (longer wavelength)</td>
<td>(\varphi_l)</td>
<td>(-\Omega_{ee})</td>
</tr>
<tr>
<td>Normalized nonlinear susceptibility (shorter wavelength)</td>
<td>(c_s)</td>
<td>(9.41 \times 10^{-22})</td>
</tr>
<tr>
<td>Normalized nonlinear susceptibility (longer wavelength)</td>
<td>(c_l)</td>
<td>(9.14 \times 10^{-22})</td>
</tr>
<tr>
<td>Normalized nonlinear susceptibility (THz wavelength)</td>
<td>(c_r)</td>
<td>(8.22 \times 10^{-4})</td>
</tr>
</tbody>
</table>

In this chapter the results of numerical simulation for normalized equations are presented. We calculated time series and power spectra for different values of time delay and feedback strength. Values of the other system parameters are given in Tab.2. In the modeling we took: \(T = 100, \mu = 0.01, \alpha = 4\). In the simulation we use relatively small values of the feedback strength considering special features of the Lang-Kobayashi model. Several characteristic regimes are computed and plotted. At \(K = 0.01\) and \(\tau = 5\) \((\varphi_s = -0.50 \times 10^4, \varphi_l = -0.45 \times 10^4)\) there single non-zero steady point exists in the phase space, i.e., output THz intensity level does not depend on time (Fig.2a). With the increase of time delay at \(\tau = 20\) \((\varphi_s = -2.00 \times 10^4, \varphi_l = -1.80 \times 10^4)\) real parts of pair of rightmost roots of characteristic equations obtain positive real parts. The point undergoes Andronov-Hopf type bifurcation and loses stability. As a result, periodic oscillation starts in the system (Fig.2b). Eigenvalues were calculated using DDE-BIFTOOL.\(^{26}\) Further increase of time delay reveals no characteristic regimes beside mentioned above. Steady state and periodic solutions interlace with the increase of time delay. This fact corresponds to different external cavity modes (ECM) excitation as a result of controlling parameters variation. These modes are the solutions of the LK equations with constant amplitude, frequency and carrier number. ECM can be found analytically from system (3).\(^{27}\) At \(K = 0.1\) and \(\tau = 5\) \((\varphi_s = -0.50 \times 10^4, \varphi_l = -0.45 \times 10^4)\) there is also single steady state point (Fig.3a). Increasing value of time delay results in excitation of self-oscillation at \(\tau = 20\) \((\varphi_s = -2.00 \times 10^4, \varphi_l = -1.80 \times 10^4)\) (Fig.3b). If one increases \(\tau\) further stable period-1 orbit undergoes period-doubling bifurcation and gives rise to stable period-2 orbit at \(\tau = 21\) \((\varphi_s = -2.10 \times 10^4, \varphi_l = -1.89 \times 10^4)\) (Fig.3c). The system also demonstrates chaotic behavior \((\tau = 22, \varphi_s = -2.20 \times 10^4, \varphi_l = -1.98 \times 10^4)\) (Fig.3d). Chaotic regime is formed by unstable periodic orbits. This regime is of particular importance for operating a system of two coupled lasers. First, two coupled synchronized lasers in chaotic regime can be used to implement secure communication. Communication scheme based on these type of synchronization of is faster, cheaper and can utilize the existing telecommunication infrastructure for these type of laser.\(^{28}\) Second, output intensity level at certain moments of time is far above the average level. Thus the system in chaotic regime can be used as a pulse generator.
4. CONCLUSION

We present the dynamic model of two-color vertical external cavity surface emitting laser with nonlinear crystal inside the cavity for THz generation. At certain values of controlling parameters only steady state solution exists, that is, output THz intensity level does not depend on time. However, varying control parameters can give rise to periodic and chaotic behavior of output intensity. Numerical experiment was carried out with the pump rate slightly above the threshold. One should expect that increasing of the pump rate will give rise to new characteristic regimes and more complex dynamic.

REFERENCES


Figure 1. The model of dual-wavelength VECSEL with intracavity nonlinear frequency conversion.
Figure 2. Time series and power spectra for $K = 0.01$ and $\tau = 5(a), \tau = 20(b)$

Figure 3. Time series and power spectra for $K = 0.1$ and $\tau = 5(a), \tau = 20(b), \tau = 21(c), \tau = 22(d)$